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The D0-D8 system revisited

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Abstract: We address a few subtle issues regarding the interacting D0-D8 system. There are two existing interpretations for the counter-intuitive non-vanishing cylinder-diagram R-R potential. We improve them each by properly dealing with the divergence of potential in the R-R as well as the NS-NS sector, which has been ignored so far. We further test them by considering the D8 to carry a flux, electric or magnetic. We find that the improved interpretations continue to hold. We resolve a subtle issue regarding the regularization of fermionic zero-modes in the R-R sector when the D8 carries an electric flux so that a meaningful result for the potential can be calculated. The persistence of divergence for the potential in either sector in the presence of a flux on the D8 brane indicates that adding a flux/fluxes on the D8 brane doesn't help to improve its nature of existence as an independent object, therefore reenforcing the previous assertion on D8 branes.

Keywords: D-branes, Supersymmetry Breaking

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Contents

1 Introduction

Among various Dp branes with different dimensionality p, the D8 brane is peculiar [1–3] and difficult to understand: as a codimension one extended object, its Ramond−Ramond (R-R) field does not fall off with distance (like a planar source in 3 + 1 dimensions). As such, when its bulk description is considered in terms of dilaton and metric, the dilaton diverges a finite distance from the brane. It is thus believed that the D8-brane cannot exist as an independent object, but only in connection with orientifold planes, for example, arising in the T-dual of the type I theory [1].

Part of the above nature of D8 brane also manifests itself in the interacting D0−D8 system. For example, from the closed string viewpoint, the stringy cylinder-diagram interaction energy from the Neveu-Schwarz−Neveu-Schwarz (NS-NS) sector (also from the R-R sector) is always divergent. On the one hand, this system, like ND = 4 system,[1] preserves 1/4 bulk supersymmetries (susy) and as such the net static interaction between the two branes vanishes due to the "abstruse identity". The zero net interaction for the ND = 4 case is, however, due to the separately vanishing contribution from either the NS-NS sector or the R-R sector. The vanishing NS-NS sector contribution is expected since the two D-branes involved carry different R-R charges, therefore giving an expected zero R-R sector contribution. However, for the ND = 8 case at hand, we actually have an infinite NS-NS contribution and this must imply an infinite R-R contribution which cancels precisely the NS-NS contribution. One special feature for this particular system is that only

[1]For an open string stretched between a Dp and a Dp'(assuming $p \geq p'$), we have the string to satisfy for every coordinate a Neumann (N) or Dirichlet (D) boundary condition at either end. We denote by NN the number of coordinates for which both ends have a N condition, by DD the number of coordinates for which both ends have a D condition, and by ND the number of coordinates for which one end has a N condition while the other has a D condition. Here we have the number of mixed boundary conditions ND $= p - p'$. A zero net static force occurs for ND = 0, 4, and 8, respectively [4–6]. For the D0-D8 system, we have ND = 8 and NN = DD = 1. In this paper, for each given brane, we use x^α to label the coordinates along the N-directions and y^i to label the coordinates along the D-directions.

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the massless modes rather than the full string spectrum appear to contribute in either sector. The contribution from the R-R sector is however puzzling and counter-intuitive since one naively does not expect a D0 and a D8 to interact through a R-R gauge field.[2]

For this, there were various efforts trying to understand the puzzle behind [7–9]. One way to interpret this massless contribution is to identify it as due to a string stretched between the two branes with its tension one-half of that of a fundamental string [7, 8]. This is supported by the existence of a coupling of a string, with its tension one-half of that of the fundamental string, with a worldvolume gauge field in the 8-brane effective action in the presence of a D0 brane [7]. This is further supported when the 0-brane in the presence of the 8-brane is considered in the massive type IIA supergravity along the line of [10]. This interpretation explains the anomalous creation of a fundamental string when the 0-brane crosses the 8-brane and this string creation is in turn related to the Hanny-Witten effect [11] by a series of dualities. It holds also for system D_{8-p}-D_p which is related to the D0-D8 system by T-duality. This approach considers so far only the finite piece of the infinite contribution from either the NS-NS or the R-R sector.

The other interpretation tries to follow the same footing as in ND $= 0$ and 4 cases. This is to identify the R-R charge carried by the 0-brane with the one opposite to the R-R charge carried by the 8-brane via a duality relation which holds only for this particular system and those related to this by T-duality and as such the interaction from the R-R sector is nothing but the usual attractive Coulomb-like force between the two branes [9].

In this paper, we try to address how to implement these interpretations with the consideration of the divergence of the potential and extend them to the case when the D8 brane carries a constant flux, electric or magnetic, along with a few subtle issues which need clarifications. We also examine whether there is an improvement on the D8 brane nature of existence as an independent object when it carries a flux, electric or magnetic.

This paper is organized as follows. In section 2, we address an issue on how to deal with the divergence in the interaction energy from either sector between a D0 and a D8 which has been ignored so far in the first interpretation even though this is not a real concern in the above second interpretation. We also calculate the R-R Coulomb potential including the divergent piece via an effective field theory approach where the crucial duality relation recognized in the second interpretation is employed. In section 3, we consider the case of D8 carrying a constant electric flux and show that the improved versions of these interpretations continue to hold. We also address a related subtle issue regarding the regularization of fermionic zero-modes in the calculation of the R-R contribution. In section 4, we consider the case of D8 carrying a magnetic flux and show that once again the improved versions still hold even though there is now a net force between the two branes. We also examine if the D8 brane nature of existence as an independent object can be improved when it carries a flux, electric or magnetic. We summarize the results in section 5.

[2]In the open string description, this contribution is from the R $(-1)^F$ sector since the two fermionic zero modes in one NN and one DD directions cancel the superghost zero modes but the contribution from the NS $(-1)^F$ sector vanishes due to the fermionic zero modes along eight ND-directions.

2 On the divergence

For the system under consideration and for concreteness, we assume the D0 along x^0 and the D8 along $x^0, x^1, \cdots x^8$ with the D0 located at $y^1 = y^2 = \cdots = y^8 = 0, y^9 = -Y$ with $Y > 0$(see footnote 3 for notation conventions) and the D8 at $y^9 = 0$. In other words, the two are separated by a distance Y along the 9th direction. The interaction potential between the two in either sector can be calculated either as a one-loop annulus amplitude from the open string description [6] or as a tree-level cylinder amplitude from the closed string boundary state description [9]. The contribution of the NS and R open string sectors to the potential per unit D0 brane worldvolume in the NS-NS sector is

$$V_{\mathrm{NS-NS}}(Y) = \frac{1}{2}\,(8\pi^2\alpha')^{-1/2}\int_0^\infty dt\, e^{-\frac{Y^2 t}{2\pi\alpha'}}\, t^{-3/2}, \tag{2.1}$$

which gives rise to a finite constant repulsive force acting on the D0 as

$$F_{\mathrm{NS-NS}} = \frac{dV_{\mathrm{NS-NS}}}{dY} = -\frac{1}{(4\pi)^{1/2}}\frac{1}{2\pi\alpha'}\int_0^\infty dx\, e^{-x}\, x^{-1/2} = -\frac{1}{4\pi\alpha'} = -\frac{T_0}{2}, \tag{2.2}$$

where we have changed the integration variable t to $x = Y^2 t/2\pi\alpha'$ in the second equality and the resulting integration is simply the well-defined gamma-function $\Gamma(1/2)$, and α' is the string constant, related to the tension of a fundamental string via $T_0 = (2\pi\alpha')^{-1}$. The contribution of R $(-1)^F$ open string sector to the cylinder potential per unit D0-brane worldvolume in the R-R sector is

$$V_{\mathrm{R-R}}(Y) = -\frac{1}{2}\,(8\pi^2\alpha')^{-1/2}\int_0^\infty dt\, e^{-\frac{Y^2 t}{2\pi\alpha'}}\, t^{-3/2}, \tag{2.3}$$

which gives rise to an finite attractive constant force acting on the D0 brane. Similarly, this force can be obtained as

$$F_{\mathrm{R-R}} = \frac{dV_{\mathrm{R-R}}}{dY} = \frac{T_0}{2}. \tag{2.4}$$

The peculiar feature for this particular system is that only the massless modes rather than the full string spectrum appear to contribute to the potential in either sector. In addition, the force from either sector is constant as shown above. As expected, due to the preservation of 1/4 susy, the total potential or the net force acting on either object in the system vanishes as obviously from the above. The non-vanishing R-R contribution can either be inferred from the 'no-force' condition with the easily calculated NS-NS contribution for this system or can be calculated, using a regularization of fermionic zero-modes, from the one-loop open string annulus diagram in the $R(-1)^F$ sector [6] or from the tree-level closed string cylinder diagram in the R-R sector [9]. In spite of this, it is puzzling and difficult to understand this result as discussed in the Introduction.

To resolve this puzzle, one way is to interpret the constant attractive R-R force given in eq. (2.4) above as due to a string stretched between the D0 and the D8 with its tension one half of that of a fundamental string [7, 8]. This interpretation is in line with [10] for a D0 brane in the presence of a D8 brane and is also consistent with the Hanny-Witten

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effect [11] for a D0 crossing a D8. However, there is a subtle issue regarding the divergence of potential in either sector given above which has been ignored so far, for example, in [8] when such an interpretation is invoked. The potential in either sector as given above contains the following integration

$$\int_0^\infty dt\, e^{-\frac{Y^2 t}{2\pi\alpha'}}\, t^{-3/2} = \frac{Y}{(2\pi\alpha')^{1/2}} \int_0^\infty dx\, e^{-x}\, x^{-3/2}, \tag{2.5}$$

which is actually divergent and where on the right we have used the integration variable $x = Y^2 t/2\pi\alpha'$. Note that the integration on the right above is not simply the gamma function $\Gamma(-1/2)$ as usually taken in the literature, for example, in [8]. The integral representation of $\Gamma(z)$ with z a complex number

$$\Gamma(z) = \int_0^\infty dx\, e^{-x} x^{z-1}, \tag{2.6}$$

is valid only for $\mathrm{Re}\,(z) > 0$. When $\mathrm{Re}\,(z) \leq 0$, $\Gamma(z)$ can still be defined but it doesn't have the above integral representation. One has to use its other representations. In other words, the integration on the right of eq. (2.5) is actually divergent, not the finite $\Gamma(-1/2)$. Due to the divergent nature, we need to deal with the integration carefully. For this let us begin with the left side of (2.5) and denote the integration as I

$$I \equiv \int_0^\infty dt\, e^{-\frac{Y^2 t}{2\pi\alpha'}}\, t^{-3/2} = -2\, t^{-1/2}\, e^{-\frac{Y^2 t}{2\pi\alpha'}}\Big|_0^\infty - \frac{2Y}{(2\pi\alpha')^{1/2}} \int_0^\infty dx\, e^{-x}\, x^{-1/2}, \tag{2.7}$$

where we have performed an integration by part and changed the integration variable to $x = Y^2 t/2\pi\alpha'$ in the second term. The first term is ∞, denoted as I_∞. The integration in the second term is nothing but the well-defined $\Gamma(1/2) = \sqrt{\pi}$. Note that the divergence of I_∞ is due to $t = 0$ at which the exponential in the first term becomes unity and we therefore expect I_∞ to be independent of Y. This is consistent with the fact that the force calculated in either sector above is constant and can indeed be obtained merely from the second term. So we have now $I = I_\infty - \frac{2\pi^{1/2} Y}{(2\pi\alpha')^{1/2}}$, i.e., a separation-independent divergent term plus a separation-dependent finite piece. With this, we have

$$V_{\mathrm{NS-NS}}(Y) = \frac{I_\infty}{2(8\pi^2\alpha')^{1/2}} - \frac{Y}{4\pi\alpha'}, \qquad V_{\mathrm{R-R}}(Y) = -\frac{I_\infty}{2(8\pi^2\alpha')^{1/2}} + \frac{Y}{4\pi\alpha'}. \tag{2.8}$$

So in addition to the finite piece, the potential in each sector has a divergent piece which is independent of the brane separation. It is obvious that this divergent piece is the contribution between the D0 and the D8 when the two is put on top of each other, i.e., with a zero separation. So this piece in either sector can be viewed as the corresponding zero-point energy. When the two move away from each other, there is an additional finite piece created which can be viewed as the Casimir energy of the system in either sector. In the R-R sector, this can be taken as due to the creation of a string stretched between the two with its tension one half of that of the fundamental string. Note that the divergence must come from the nature of D8 brane as mentioned in the Introduction. So whenever this persists, the corresponding system cannot be taken as an independent object.

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With this divergent piece in either sector, how then can we make a consistent picture using the creation of a fundamental string when the D0 crosses the D8 adiabatically? For this, we need to use the improved version of this interpretation given in [12] where it is stressed that such a crossing is merely a parity transformation along the DD direction (i.e., along the x^9 direction). Such a transformation will reverse the sign of $R(-)^F$ term but leave the rest of terms invariant, therefore not converting a brane into an anti-brane in general even though it doesn't make difference for the present case since we have the $NS(-)^F$ term vanishing. If the initial configuration is supersymmetric as the present case and is taken as the vacuum configuration, we have the total potential as $NS + R + NS(-)^F + R(-)^F$, where the first two terms give the NS-NS sector potential while the last two terms give the R-R sector one. After the adiabatically crossing, we end with a vacuum configuration $NS + R + NS(-)^F - R(-)^F$ since only the $R(-)^F$ will change its sign, which is obviously non-supersymmetric unless the $R(-)^F$ term vanishes. But this is an adiabatic process and the total energy should be kept unchange, therefore we must have created an additional term $2R(-)^F$ in the process, as argued in [12], such that

$$NS + R + NS(-)^F + R(-)^F = \left(NS + R + NS(-)^F - R(-)^F\right) + 2R(-)^F. \tag{2.9}$$

If the divergent piece in either sector is not considered, then the above is consistent with the creation of a fundamental string since the term $2R(-)^F$ is just the tension of a fundamental string times the separation. For the present case, the above continues to hold if we include the divergent piece in either sector and the interpretation is now that the crossing creates not only a fundamental string but also twice the $R(-)^F$ sector zero-point energy before crossing whose origin may be better explained using the second interpretation proposed in [9] when the brane separation vanishes. So we further improve the improved interpretation proposed in [12] here.

Note that we don't need to improve the second interpretation given in [9] much except for merely insisting that the Coulomb-like potential includes the divergent piece as well as the finite piece (so does the potential in the NS-NS sector) for which we turn next. This R-R interaction is purely due to massless modes and as mentioned earlier, just like the field theory limit of the ND $= 0$ case, it is simply due to the usual Coulomb-like force between the two D-branes. This is however counter-intuitive for the ND $= 8$ system and any other system related to this by T-duality. The complete discussion given [9] in terms of relevant boundary states and vertex operators is lengthy and here we will use the effective field theory approach, for example, following [13], to derive this same interaction based on the underlying physics and the duality relation given there. The key for this is the recognition that the usually completely decoupled unphysical degrees of freedom such as the longitudinal and scalar states, the analogues of the scalar and longitudinal photons of electrodynamics, in perturbative string theory becomes important when dealing with the non-perturbative boundary states. This consideration will give rise to the duality relation, mentioned earlier, which is crucial for the interaction to arise.

Since only the massless modes in type IIA are relevant here, the interaction in the R-R sector is due to the exchange of an off-shell closed string between the two R-R charges, just like the usual Coulomb interaction between two static charges as due to the exchange

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of a virtual photon. As discussed in [9], this off-shell closed string state can be represented by the corresponding vertex operator which can be constructed from the on-shell massless closed string vertex operator and by allowing the momentum $k^2 \neq 0$, i.e., off-shell. As described in [9], the R-R sector boundary state used in the cylinder-diagram calculation is in the $(-1/2, -3/2)$ picture and to soak up the superghost number anomaly, it can only couple to states that are also in the asymmetric picture. However, the vertex operators used usually for constructing the R-R states are in the symmetric $(-1/2, -1/2)$ picture and are given as

$$V_R(k; z, \bar{z}) = \frac{(C\Gamma^{\mu_1\mu_2\cdots\mu_{m+1}})_{\alpha\dot{\beta}}}{2\sqrt{2}(m+1)!} F_{\mu_1\mu_2\cdots\mu_{m+1}} V^{\alpha}_{-1/2}(k/2; z) \tilde{V}^{\dot{\beta}}_{-1/2}(k/2; \bar{z}), \tag{2.10}$$

with m odd (even) in IIA (IIB) theory and $V^{\alpha}_{l}(k; z) = c(z) S^{\alpha}(z) e^{l\phi(z)} e^{ik\cdot X(z)}$. The $(m+1)$-form F_{m+1} gives just the R-R field strength of IIA or IIB theory since the BRST invariance of the vertex operator requires $k^2 = 0$, and $dF_{m+1} = d * F_{m+1} = 0$ which are precisely the Bianchi identity and the field equations of motion for the field strength.

The vertex operators in the $(-1/2, -3/2)$ can also be constructed and they can be transformed to the ones in the symmetric picture via a picture changing operation in the right sector [14]. For the present purpose, we don't need the complete construction as given in [9] but the following term [15]

$$W^{(0)}(k; z, \bar{z}) = \frac{(C\Gamma^{\mu_1\cdots\mu_m})_{\alpha\dot{\beta}}}{m!} A_{\mu_1\cdots\mu_m} V^{\alpha}_{-1/2}(k/2; z) \tilde{V}^{\dot{\beta}}_{-3/2}(k/2; \bar{z}). \tag{2.11}$$

Note that instead of the R-R field strength as in the symmetric picture, we need here only the R-R m-form gauge potential A_m. The BRST invariance of this vertex requires $k^2 = 0$ and $dA_m = d * A_m = 0$, therefore just a pure gauge potential. But this is sufficient for what follows.

In the symmetric case, the electric-magnetic duality is $F_{m+1} \simeq *F_{9-m}$. If we consider the spatial momentum in (2.10) only along the 9th direction, then we have from the duality relation

$$F_{01} = k_0 A_1 = -F_{23\cdots 9} = k_9 A_{2\cdots 8}. \tag{2.12}$$

Since the state considered is massless, therefore $k_0 = k_9$ and this gives

$$A_1 = A_{2\cdots 8}. \tag{2.13}$$

It is important to realize that the above relation involves only the physical degrees of freedom.

We now move to the asymmetric state (2.11). As will be seen, the above given conditions for keeping the state BRST invariant are precisely those fulfilled by the mixture of longitudinal and scalar polarizations which describe the Coulomb interaction. Let us focus first on the 1-form potential and consider again the spatial momentum along 9th direction. Then the pure gauge solution is simply, with $k_0 = k_9$, as

$$A_0 = A_9, \quad \text{and} \quad A_i = 0, \quad \text{for} \quad i = 1, 2, \cdots 8. \tag{2.14}$$

For the state (2.11), we have a Hodge duality for the 10-dimensional unphysical polarizations. For 1-form potential, we have $A_9 = -A_{01\cdots 8}$. When combined this with (2.14), we have

$$A_0 = -A_{01\cdots 8}. \tag{2.15}$$

This unusual relation is of no relevance in perturbative string theory where the unphysical degrees of freedom always decouple but it has remarkable consequences for the present case. In fact, it implies that the charge felt by A_0 is opposite to that felt by $A_{01\cdots 8}$ and thus the attractive Coulomb R-R force between a D0 and a D8 brane can be understood as due to the exchange of longitudinal and scalar polarizations identified according to (2.15). In the following, we will use this relation to derive the R-R Coulomb interaction via the field theory approach [13].

With the canonical normalization [13] for the background R-R potential, the coupling for a D0 with the 1-form R-R potential A_0 is

$$J^{(0)} = \sqrt{2}\, c_0\, V_1\, A_0, \tag{2.16}$$

and the coupling for a D8 with the 9-form R-R potential is

$$J^{(8)} = \sqrt{2}\, c_8\, V_9\, A_{01\cdots 8}, \tag{2.17}$$

where V_{p+1} is the worldvolume of D_p brane and the constant $c_p = \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-p}$. Then the static interaction per unit D0 brane worldvolume due to the massless modes in the R-R sector can be calculated in the momentum space simply as

$$V_{\mathrm{R-R}}(k_\perp) = -\frac{1}{V_1 V_9}\underbrace{J^{(0)} J^{(8)}} = -2\, c_0\, c_8 \underbrace{A_0\, A_{01\cdots 8}} = -\frac{1}{2\pi\alpha'\, k_\perp^2}, \tag{2.18}$$

where we have used the explicit expression of c_p and the key fact

$$\underbrace{A_0\, A_{01\cdots 8}} = -\underbrace{A_0\, A_0} = -\underbrace{A_{01\cdots 8}\, A_{01\cdots 8}} = \frac{1}{k_\perp^2} \neq 0, \tag{2.19}$$

because of (2.15). In the above, the propagator $\underbrace{A_{0\cdots p}\, A_{0\cdots p}} = -1/k_\perp^2$ is used and the $k_\perp$ is the spatial momentum perpendicular to both branes, i.e., the momentum along the 9-th direction for the present case. So the potential in coordinate space is

$$\begin{aligned} V_{\mathrm{R-R}}(Y) &= \int_{-\infty}^{\infty} \frac{dk_\perp}{2\pi} e^{-ik_\perp Y} V_{\mathrm{R-R}}(k_\perp) \\ &= -\frac{2}{(2\pi)^2\alpha'}\int_0^\infty dk_\perp \frac{\cos k_\perp Y}{k_\perp^2} \\ &= \frac{2}{(2\pi)^2\alpha'}\left(\frac{\cos k_\perp Y}{k_\perp}\Bigg|_0^\infty + Y\int_0^\infty \frac{\sin k_\perp Y}{k_\perp} dk_\perp\right) \\ &= -\frac{I_\infty}{2(8\pi^2\alpha')^{1/2}} + \frac{Y}{4\pi\alpha'}, \end{aligned} \tag{2.20}$$

where in obtaining the last line we have defined $k_\perp = \sqrt{2t/\pi^2\alpha'}$ in the first term and made use of $\int_0^\infty dx \sin x/x = \pi/2$ in the second term in the bracket in the third line above. This potential is identical to the stringy result given earlier in (2.8) but is now calculated via the field theory approach as the Coulomb interaction. As stressed, the infinite piece should be kept which reflects the nature of the D8 brane and it is just the Coulomb interacting energy when the brane separation vanishes.

3 The D8 with an electric flux

In this section, we will examine whether the improved versions of the two interpretations for the non-vanishing R-R potential discussed in the previous section in the absence of a flux continue to hold when the D8 carries an electric flux. We will also discuss a subtle issue regarding the regularization of fermionic zero-modes in the calculation of the R-R potential for the present case.

For this, we first calculate the potential in the NS-NS sector. This can be worked out in almost the same way as the case when the D8 doesn't carry any flux [6, 9] if a trick as adopted in [13, 16, 17] is employed. In the following, we will perform the calculations using the closed string operator formalism in which D-branes with/without constant fluxes can be described by the so-called boundary states. In this approach, there are two sectors, namely NS-NS and R-R sectors, respectively. For later convenience, we give a brief description of each and refer to [9, 13, 16] for detail.

Both in the NS-NS and R-R sectors, there are two possible implementations for the boundary conditions of a D-brane which correspond to two boundary states $|B,\eta\rangle$ with $\eta=\pm$. However, only the so-called Gliozzi-Scherk-Olive (GSO) combinations $|B\rangle_{\rm NS-NS} = \frac{1}{2}\left[|B,+\rangle_{\rm NS-NS} - |B,-\rangle_{\rm NS-NS}\right]$ and $|B\rangle_{\rm R-R} = \frac{1}{2}\left[|B,+\rangle_{\rm R-R} + |B,-\rangle_{\rm R-R}\right]$ are selected in the NS-NS and in the R-R sectors, respectively. The boundary state $|B,\eta\rangle$ is the product of a matter part and a ghost part as $|B,\eta\rangle = \frac{c_p}{2}|B_{\rm mat},\eta\rangle|B_{\rm g},\eta\rangle$ with $|B_{\rm mat},\eta\rangle = |B_X\rangle|B_\psi,\eta\rangle$, $|B_{\rm g},\eta\rangle = |B_{\rm gh}\rangle|B_{\rm sgh},\eta\rangle$ and the overall normalization $c_p = \sqrt{\pi}(2\pi\sqrt{\alpha'})^{3-p}$. The boundary states for ghosts and superghosts are independent of the fluxes and therefore remain the same as before. We will not list them here for simplicity. The expressions of the matter part of $|B,\eta\rangle$ are given, respectively, as $|B_X\rangle = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\cdot S\cdot\tilde{\alpha}_{-n}\right]|B_X\rangle^{(0)}$, and $|B_\psi,\eta\rangle_{\rm NS-NS} = -{\rm i}\exp\left[{\rm i}\,\eta\sum_{m=1/2}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|0\rangle$ for the NS-NS sector, and $|B_\psi,\eta\rangle_{\rm R-R} = -\exp\left[{\rm i}\,\eta\sum_{m=1}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|B,\eta\rangle^{(0)}_{\rm R-R}$ for the R-R sector. They look the same in form as their correspondences in the absence of fluxes. All the information about the constant flux F is encoded in the S-matrix

$$S = \left(\left[(\eta-\hat{F})(\eta+\hat{F})^{-1}\right]_{\alpha\beta}, -\delta_{ij}\right), \tag{3.1}$$

and the zero-mode boundary states: for the bosonic sector as

$$|B_X\rangle^{(0)} = \sqrt{-\det\left(\eta+\hat{F}\right)}\,\delta^{9-p}(q^i-y^i)\prod_{\mu=0}^{9}|k^\mu=0\rangle, \tag{3.2}$$

and for the R sector as

$$|B_\psi,\eta\rangle^{(0)}_{\rm R} = \left(C\Gamma^0\Gamma^1\cdots\Gamma^p\frac{1+{\rm i}\,\eta\Gamma_{11}}{1+{\rm i}\,\eta}U\right)_{AB}|A\rangle|\tilde{B}\rangle. \tag{3.3}$$

In the above, we have denoted by y^i the positions of the D-brane along the transverse directions, by C the charge conjugation matrix and by U the following matrix

$$U = \frac{1}{\sqrt{-\det(\eta + \hat{F})}} ; \exp\left(-\frac{1}{2}\hat{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta\right); \tag{3.4}$$

where the symbol ; ; means that one has to expand the exponential and then to anti-symmetrize the indices of the Γ-matrices. $|A\rangle|\tilde{B}\rangle$ stands for the spinor vacuum of the R-R sector. We also define $\hat{F} = 2\pi\alpha' F$ in the above. We would like to point out that the η in the above means either sign $\pm$ or the flat signature matrix $(-1, +1, \cdots, +1)$ on the world-volume and should not be confused from the content.

Note that D0 cannot carry any flux and without loss of generality, we can always choose the constant electric flux on the D8 with the only non-vanishing components $(\hat{F})_{01} = -(\hat{F})_{10} = -f$. We then have the S-matrix for D0 and D8, respectively, as

$$S^0_{\mu\nu} = -\delta_{\mu\nu}, \quad S^8_{\mu\nu} = \left(S^8_{\alpha\beta}\,, -1\right), \tag{3.5}$$

with the only non-vanishing components of $S^8_{\alpha\beta}$ as $S^8_{00} = -S^8_{11} = -\frac{1+f^2}{1-f^2}, S^8_{01} = -S^8_{10} = -\frac{2f}{1-f^2}$, and $S^8_{aa} = 1$ where $a = 2, 3 \cdots 8$. With the above preparation and by considering the relevant conventions described in footnote 3, the tree-level cylinder-diagram amplitude in the NS-NS sector can be carried out straightforwardly and is given as

$$\begin{aligned}
\Gamma_{\text{NS-NS}} &= {}_{\text{NS-NS}}\langle B^0|D|B^8\rangle_{\text{NS-NS}}, \\
&= \frac{\sqrt{1-f^2}}{2}\, n_0\, n_8\, V_1\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}} \left[\frac{\theta_4^4\,(0|\,\mathrm{i}\,t)}{\theta_2^4\,(0|\,\mathrm{i}\,t)} - \frac{\theta_3^4\,(0|\,\mathrm{i}\,t)}{\theta_2^4\,(0|\,\mathrm{i}\,t)}\right], \\
&= -\frac{\sqrt{1-f^2}}{2}\, n_0\, n_8\, V_1\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}},
\end{aligned} \tag{3.6}$$

where D is the standard closed string propagator and in the last step we have used the usual Jacobi's "abstruse identity" $\theta_3^4\,(0|\,\mathrm{i}\,t) - \theta_4^4\,(0|\,\mathrm{i}\,t) - \theta_2^4\,(0|\,\mathrm{i}\,t) = 0$. In the above, we have replaced the normalization constant c_k $(k = 0, 8)$ by $n_k c_k$ with n_k an integer to count the multiplicity of the corresponding branes. In carrying out the above calculations, as mentioned at the beginning of this section, we follow the trick as adopted in [13, 16, 17] by making a respective unitary transformation of the oscillators in the boundary state for the D0 such that its S^0-matrix completely disappears while the boundary state for the D8 ends up with a new $S = S^8(S^0)^T$ where T denotes the transpose. This new S-matrix shares the same property as the original S^k satisfying $((S^k)^T)_\mu{}^\rho(S^k)_\rho{}^\nu = \delta_\mu{}^\nu$ with $k = 0\,\text{or}\,8$ but its determinant is always unity and therefore can always be diagonalized to give its eigenvalues. With this trick, the amplitude calculations as mentioned earlier are no more complicated than the case for the D8 carrying no flux.

The above amplitude gives a repulsive potential per unit D0 brane worldvolume as

$$V_{\text{NS-NS}} = \frac{\sqrt{1-f^2}}{2}\, n_0\, n_8\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}, \tag{3.7}$$

which differs from the one for the D8 carrying no flux only by an overall factor $\sqrt{1-f^2}$. Apart from this, the electric flux on the D8 doesn't change the structure of the potential at all. This is simply due to the fact that the eigenvalues of the above mentioned S-matrix, which determine the amplitude structure, remain the same as if there is no such an electric flux present. In fact, the eigenvalues of the matrix $S_\mu{}^\nu$ have two of $+1$ and eight of -1 as one can check easily and explicitly. This overall factor arises from the overall factor appearing in the bosonic zero-mode boundary state given in (3.2).

The underlying physics of the above can be understood in the following: If we T-dualize along the electric flux direction on the D8, then the D8 will become a D7 moving with a velocity of magnitude $|f|$ (≤ 1) in the original flux direction while the D0 becomes a D1 along the original flux direction but at rest. We can make a boost $\gamma = 1/\sqrt{1-f^2}$ against the D1 direction such that the D7 becomes at rest. Note that this boost has no influence on the D1 since it has a Lorentz symmetry on its worldvolume. So after the boost, both the D1 and the D7 are at rest. But such a boost has an effect on the tension and the R-R charge of the D7, reducing each by a factor of $\sqrt{1-f^2}$, since such a boost is in the direction orthogonal to its worldvolume. Now the D1 and the D7 are orthogonal to each other and are related to the D_{8-p}-D_p system by T-duality, which preserves 1/4 of supersymmetries. In particular, we can T-dualize the above D1-D7 back to a static D0-D8 system with the D8 carrying no flux but with its tension and R-R charge $\sqrt{1-f^2}$ times those of a fundamental D8, respectively. Such an understanding gives not only a physical explanation to the above calculated amplitude but also predicts that the D0-D8 system preserves the same 1/4 of susy whether the D8 carries an electric flux or not. This latter must imply, due to the 'no-force' condition, the amplitude in the R-R sector,

$$\Gamma_{\mathrm{R-R}} = \frac{\sqrt{1-f^2}}{2}\, n_0\, n_8\, V_1\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}, \tag{3.8}$$

and the corresponding attractive potential per unit D0 brane worldvolume

$$V_{\mathrm{R-R}}(Y) = -\frac{\sqrt{1-f^2}}{2}\, n_0\, n_8\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}. \tag{3.9}$$

Once again, only the massless modes rather than the full spectrum contribute to the interaction in either sector.

With the above known answer, we now try to calculate the amplitude in the R-R sector directly and correctly. The complication arises from the regularization of fermionic zero-modes which requires care. The tree-level cylinder-diagram amplitude in this sector can be similarly calculated up to

$$\begin{aligned}\Gamma_{\mathrm{R-R}} &= {}_{\mathrm{R-R}}\langle B^0|D|B^8\rangle_{\mathrm{R-R}}\\ &= \frac{\sqrt{1-f^2}}{2^5} n_0\, n_8\, V_1\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}\; {}^{(0)}_{\mathrm{R-R}}\langle B^0, \eta^0|B^8, \eta^8\rangle^{(0)}_{\mathrm{R-R}},\end{aligned} \tag{3.10}$$

where ${}^{(0)}_{\mathrm{R-R}}\langle B^0, \eta^0|B^8, \eta^8\rangle^{(0)}_{\mathrm{R-R}} = {}^{(0)}_{\mathrm{R-R}}\langle B^0_\psi, \eta^0|B^8_\psi, \eta^8\rangle^{(0)}_{\mathrm{R-R}}\; {}^{(0)}_{\mathrm{R-R}}\langle B^0_{\mathrm{sgh}}, \eta^0|B^8_{\mathrm{sgh}}, \eta^8\rangle^{(0)}_{\mathrm{R-R}}$. In extracting a finite meaningful result from the two divergent matrix elements due to the fermionic zero-modes and the superghosts, respectively, we need to regularize both properly. The regularization adopted in [8, 9] is however not directly applicable at present since the matrix U (3.4) appearing in the R-R zero-mode boundary state (3.3) in the presence of an electric flux mixes a NN-direction Γ^0-matrix with a ND-direction Γ^1-matrix. As a result, it is not possible to group pairs of Γ-matrices purely from the ND-directions and purely from NN or DD directions. In other words, if we try to group, there is at least one pair with one Γ-matrix from a ND-direction and the other from a NN- or DD-direction. Such a grouping cannot give a meaningful result as stressed in [9]. One simple reason is that this grouping is not consistent with T-duality since the corresponding SO(2) rotation can convert a ND-direction to a NN or DD-direction and vice-versa but T-duality doesn't allow a ND-direction to become a NN- or DD-direction and vice-versa.

So we face a dilemma here. The evaluation of ${}_{\mathrm{R-R}}^{\ (0)}\langle B^0, \eta^0 | B^8, \eta^8\rangle^{(0)}_{\mathrm{R-R}}$ ends up with two terms $\mathrm{Tr}(\Gamma^0\Gamma^9) - f\,\mathrm{Tr}(\Gamma^1\Gamma^9)$ with Tr denoting the trace, where the second term associated with the flux doesn't allow a meaningful regularization since '1' is a ND-direction while '9' is a DD-direction. Our previous discussion on how to get rid of the electric flux on the D8 via a T-duality and a boost motivates us to resolve this, however. These transformations shouldn't change the value of matrix element as we discussed above for the evaluation of the amplitude in NS-NS sector. So we can evaluate this matrix element after the above transformations. A T-duality doesn't change the direction of '1' but a boost will rotate '0' and '1'. This motivates us to define new $\tilde\Gamma^0$ and $\tilde\Gamma^1$ in terms of the old Γ^0 and Γ^1 as

$$\tilde\Gamma^0 = \frac{1}{\sqrt{1-f^2}}\left(\Gamma^0 - f\Gamma^1\right) \quad \text{and} \quad \tilde\Gamma^1 = \frac{1}{\sqrt{1-f^2}}\left(-f\Gamma^0 + \Gamma^1\right), \tag{3.11}$$

where the new $\tilde\Gamma^0$ and $\tilde\Gamma^1$ satisfy their respective own properties and further $\tilde\Gamma^0\tilde\Gamma^1 = \Gamma^0\Gamma^1$. Using the newly defined Γ-matrices, we have ${}_{\mathrm{R-R}}^{\ (0)}\langle B^0_\psi, \eta^0 | B^8_\psi, \eta^8\rangle^{(0)}_{\mathrm{R-R}} = -\delta_{\eta^0\eta^8,-}\mathrm{Tr}\left(\tilde\Gamma^0\Gamma^9\right)$, where the trace can now be regularized following the regularization procedure given in [9] since the $\tilde\Gamma^0$ is one along a NN-direction and Γ^9 is one along a DD-direction. It is then straightforward to have ${}_{\mathrm{R-R}}^{\ (0)}\langle B^0, \eta^0 | B^8, \eta^8\rangle^{(0)}_{\mathrm{R-R}} = 2^4$.

So we have from (3.10)

$$\Gamma_{\mathrm{R-R}} = \frac{\sqrt{1-f^2}}{2} n_0\, n_8\, V_1\,(8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}, \tag{3.12}$$

which is identical to the predicted one given in (3.8). This in turn shows that our above well-motivated regularization process makes sense.

The underlying physical picture given earlier in this section also makes it certain that the two improved interpretations discussed in the previous section continue to apply to the present system. This has to be true if these interpretations make sense indeed since the present system preserves the same number of susy as the system discussed in the previous section for which the D8 doesn't carry any flux. We know that the present system is equivalent to a D0-D8 system with the D8 carrying no flux but with its tension and R-R charge reduced by a factor of $\sqrt{1-f^2}$ but the corresponding quantities for the D0 remain untouched. This clearly indicates that the improved second interpretation regarding the R-R interaction as purely due to the usual attractive Coulomb-like force between the two branes [9] continues to hold. Also with the above in mind, it is obvious that the improved first interpretation holds also if one notices that the tension of the string stretched should also be reduced by this same factor since the string is along a DD-direction and its tension is energy per unit length and so it should be re-scaled by this same factor under the boost. This can also be equivalently inferred from the scaling of the D8-brane tension from either the R-R potential or the NS-NS potential plus the 'no-force' condition since it is just the finite piece described in the previous section.

4 The D8 with a magnetic flux

The D8 carrying a constant electric flux as discussed in the previous section is actually the so-called (F, D8) non-threshold bound state. The zero net force between a D0 and a (F, D8) can also be intuitively understood by noticing that the D0 doesn't interact with either constituent in the bound state. When a D8 carries a constant magnetic flux, it actually represents the so-called (D6, D8) non-threshold bound state. But now since the D0 has a net repulsive interaction with the D6 in the bound state [1], we expect that there is a repulsive non-zero net force between the D0 and the (D6, D8). So this system cannot preserve any susy in general[3] and one also expects that the interaction is no longer purely due to the massless modes as would be the case for pure D8 or for D8 carrying an electric flux. However, this latter point applies only to the NS-NS sector interaction since there is no R-R sector interaction between the D0 and the D6. We therefore expect that the R-R sector amplitude remains the same as in the absence of the magnetic flux which will be shown shortly. This implies that our improved interpretations given in section 2 continue to hold even in this case. Let us now calculate this non-vanishing interaction which can be performed even simpler than the previous case since we don't have a regularization issue here for the fermionic zero modes in the R-R sector.

We also have the S-matrix for D0 or D8 to be given in (3.5) but with now the only non-vanishing components of $S^8_{\alpha\beta}$ as $-S^8_{00} = S^8_{11} = \cdots = S^8_{66} = 1, S^8_{77} = S^8_{88} = \frac{1-f^2}{1+f^2}, S^8_{78} = -S^8_{87} = \frac{2f}{1+f^2}$. For this, we have taken the only non-vanishing components of the magnetic flux as $\hat{F}_{78} = -\hat{F}_{87} = -f$. With the same trick as mentioned in the previous section, the new matrix $S = S^8(S^0)^T$ has its eigenvalues as $\{\lambda, \lambda^{-1}, 1, 1, -1, -1, -1, -1, -1, -1\}$, with the sum of λ and λ^{-1}, needed only in the amplitude calculations, given as

$$\lambda + \lambda^{-1} = -2\frac{1-f^2}{1+f^2}. \tag{4.1}$$

We have then the amplitude in the NS-NS sector as

$$\begin{aligned}\Gamma_{\text{NS-NS}} &= {}_{\text{NS-NS}}\langle B^0|D|B^8\rangle_{\text{NS-NS}} \\ &= \frac{n_0\, n_8\, V_1}{2(8\pi^2\alpha')^{\frac{1}{2}}}\int_0^\infty dte^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}}\left[\frac{\theta_3\left(\nu|it\right)\theta_3^3\left(\frac{1}{2}|it\right)}{\theta_1\left(\nu|it\right)\theta_1^3\left(\frac{1}{2}|it\right)} - \frac{\theta_4\left(\nu|it\right)\theta_4^3\left(\frac{1}{2}|it\right)}{\theta_1\left(\nu|it\right)\theta_1^3\left(\frac{1}{2}|it\right)}\right],\end{aligned} \tag{4.2}$$

[3] In certain special cases as specified in [18], this system can be supersymmetric and we will not discuss these special cases here. See also [19, 20]

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where the parameter ν is defined via $\lambda \equiv \exp(2\pi\,\mathrm{i}\,\nu)$ with $\nu \in (0, 1/2]$ and from (4.1) we have $\sin\pi\nu = 1/\sqrt{1+f^2}$. The amplitude in the R-R sector can also be calculated as

$$\Gamma_{\mathrm{R-R}} = {}_{\mathrm{R-R}}\langle B^0|D|B^8\rangle_{\mathrm{R-R}} = \frac{1}{2} n_0\, n_8\, V_1\, (8\pi^2\alpha')^{-\frac{1}{2}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}} t^{-\frac{1}{2}}, \tag{4.3}$$

where we have employed the regularization for the fermionic zero modes given in [9] for the zero-mode matrix element, ${}_{\mathrm{R}}^{(0)}\langle B^0_\psi, \eta^0|B^8_\psi, \eta^8\rangle^{(0)}_{\mathrm{R}} = -\frac{\delta_{\eta^0\eta^8,-}}{\sqrt{1+f^2}} \mathrm{Tr}\left(\Gamma^0\Gamma^9\right)$, which meets the regularization requirement. Note that this R-R amplitude is indeed the same as that in the absence of the flux, as anticipated, again purely due to the massless modes. Therefore, the improved interpretations discussed in section 2 for the attractive potential in this sector continue to hold.

So the total amplitude $\Gamma = \Gamma_{\mathrm{NS-NS}} + \Gamma_{\mathrm{R-R}}$ is now

$$\begin{aligned}
\Gamma &= \frac{n_0\, n_8\, V_1}{2(8\pi^2\alpha')^{\frac{1}{2}}} \int_0^\infty dt e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}} \left[1 + \frac{\theta_3\left(\nu|it\right)\theta_3^3\left(\frac{1}{2}|it\right)}{\theta_1\left(\nu|it\right)\theta_1^3\left(\frac{1}{2}|it\right)} - \frac{\theta_4\left(\nu|it\right)\theta_4^3\left(\frac{1}{2}|it\right)}{\theta_1\left(\nu|it\right)\theta_1^3\left(\frac{1}{2}|it\right)}\right] \\
&= -\frac{n_0\, n_8\, V_1}{(8\pi^2\alpha')^{\frac{1}{2}}} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}} \frac{\theta_1^4\left(\frac{1}{4} - \frac{\nu}{2}|\,\mathrm{i}\,t\right)}{\theta_1\left(\nu|\,\mathrm{i}\,t\right)\theta_1^3\left(\frac{1}{2}|\,\mathrm{i}\,t\right)} \\
&= -\frac{n_0\, n_8\, V_1}{(8\pi^2\alpha')^{\frac{1}{2}}} \frac{\sin^4\pi\left(\frac{1}{4} - \frac{\nu}{2}\right)}{\sin\pi\nu} \int_0^\infty dt\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{1}{2}} \\
&\qquad\qquad \times \prod_{n=1}^\infty \frac{\left(1-\bar\lambda|z|^{2n}\right)^4\left(1-\bar\lambda^{-1}|z|^{2n}\right)^4}{\left(1-\lambda|z|^{2n}\right)\left(1-\lambda^{-1}|z|^{2n}\right)\left(1+|z|^{2n}\right)^6},
\end{aligned} \tag{4.4}$$

where in the second equality we have made use of the following fundamental Jacobian identity $-2\theta_1^4\left(\frac{\nu}{2} - \frac{1}{4}|\,\mathrm{i}\,t\right) = \theta_1\left(\nu|\,\mathrm{i}\,t\right)\theta_1^3\left(\frac{1}{2}|\,\mathrm{i}\,t\right) + \theta_3\left(\nu|\,\mathrm{i}\,t\right)\theta_3^3\left(\frac{1}{2}|\,\mathrm{i}\,t\right) - \theta_4\left(\nu|\,\mathrm{i}\,t\right)\theta_4^3\left(\frac{1}{2}|\,\mathrm{i}\,t\right)$ which can be obtained from the equation (iv) on page 468 in [21] with certain choices of variables and also a relation $\theta_1(1-\nu|it) = \theta_1(\nu|it)$. In the above $|z| = e^{-\pi t}$ and $\bar\lambda = \exp[2\pi i(1/4 - \nu/2)]$. It is obvious that the amplitude vanishes only in the absence of the flux, i.e., $\nu = 1/2$. Noticing that $\left(1-\lambda|z|^{2n}\right)\left(1-\lambda^{-1}|z|^{2n}\right) = \left(1 - 2\cos 2\pi\nu\,|z|^{2n} + |z|^{4n}\right) > 0$, this amplitude is always less than zero for the non-vanishing flux, therefore implying a repulsive interaction as anticipated. This can further be understood as follows: the interactions between the D0 and D6 and between the D0 and the D8 are both repulsive in the NS-NS sector while the interaction from the R-R sector is purely from that between the D0 and D8. Also in the absence of the D6 in the bound state, the net interaction vanishes. So now we must have a repulsive interaction since the repulsive part is enlarged.

From the above calculations, it becomes clear that adding a flux, electric or magnetic, to the D8 will not change the singular behavior of the R-R potential which is due to massless modes. This implies that if there is any change to the NS-NS potential, it must be an overall factor and/or a change on the part due to massive modes which will not be important for large t in the closed string cylinder-diagram or small t in the open string annulus diagram. Precisely in this region, the t-integration gives rise to the singularity. This will remain so even for a more general flux or fluxes. So we conclude that the non-threshold (F, D8) or (D6, D8) bound state or a D8 carrying more general flux/fluxes will not exist as an independent object just like the D8 carrying no flux.

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5 Summary

In this paper, we address various subtle issues regarding the D0-D8 system. First we show that the potential in either the NS-NS or R-R sector is actually divergent and can be expressed as a brane-separation-independent divergent piece plus a brane-separation-dependent finite piece. This divergent piece in each sector is the interaction energy when the D0 and the D8 has zero separation, therefore can be viewed as the zero-point energy in this sector. The finite piece can be viewed as the Casimir energy and in particular in the R-R sector it can be viewed as due to a string created with its tension one half of that of a fundamental string when the D0 moves away from the D8 at a distance. With this, we have improved the understanding of a fundamental string creation as given in [12] when the D0 crosses the D8 by including the divergent piece in each sector before and after the crossing. The other interpretation continues to hold if the divergent piece is also included in the R-R potential.

We have also shown that the above improved interpretations regarding the non-vanishing attractive R-R potential continue to hold even in the presence of a flux, electric or magnetic. When the flux is electric, we also resolve a subtle issue in section 3 on how to implement properly the regularization of fermionic zero-modes given in [9] so that a meaning result can be obtained for the R-R potential.

The persistence of divergence for the amplitude in each sector in the presence of a flux, electric or magnetic, or even a more general flux indicates that the non-threshold (F, D8) or (D6, D8) bound state, or the D8 carrying a more general flux cannot exist as an independent of object, either.

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